An isogeometric boundary element scheme for transient heat transfer problems in electronic packaging

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2023 24th International Conference on Electronic Packaging Technology (ICEPT) | 979-8-3503-3881-2/23/$31.00 ©2023 IEEE | DOI: 10.1109/ICEPT59018.2023.10492337

***Abstract*—With the development of electronic packaging technology, the packaging density of electronic components is increasing, and thermal issues have gradually been a main reason for electronic component failure. This paper proposes a transient isogeometric boundary element method (IGABEM) to solve the transient thermal problem in electronic packaging. The proposed method combines the advantages of isogeometric analysis (IGA) and boundary element method (BEM). Compared with finite element method, only the boundary element is needed in the computation, which reduces the preprocessing time since meshing process inside the computational domain being avoided. The radial integration method (RIM) is used to transform the domain integrals into equivalent boundary integrals. This work uses time marching method to handle equation that involves the time term. Finally, a transient heat transfer problem of a simplified chip-copper pillar bump structure is used to illustrate the accuracy of the proposed method.**

Keywords—Isogeometric boundary element method; Transient heat transfer problems; Radial integration method

I. INTRODUCTION

Electronic packaging not only provides electrical signal connection and power control, but also plays an important role in heat dissipation and mechanical protection [1]. With the continuous improvement of electronic packaging technology, electronic packaging continues to develop along the trend of light weight, multi-functionality, high performance and high integration. The characteristic size of packaging structures varies by several orders of magnitude [2], showing multi-scale features. According to statistics, the failure of most electronic products is caused by temperature [3], therefore transient heat transfer problems have become one of the main focuses of reliability research on packaging structures.

Scholars usually use the finite element method (FEM) to study the thermal problems in electronic devices. Xiao et al. [4] proposed a calculation method for the equivalent thermal conductivity of TSV intermediate layers based on finite element analysis and verified the correctness of this method through numerical simulation. To ensure the accuracy of the calculation results when studying the electronic packaging structure through the finite element method, large number of elements is needed, which increases the calculation cost. The boundary element method (BEM) is a numerical analysis method based on boundary integral equations. Its main idea is to transform the governing equations into boundary integral equations. Compared with FEM, BEM only needs to calculate the physical quantities on the boundary during numerical calculation. Thus, BEM reduces the dimensionality of the problem by one. The physical quantities related to derivatives in BEM can be obtained by differentiating the boundary integral equation. In addition, the boundary integral equation uses an analytical fundamental solution, so the accuracy of physical quantities related to derivatives in BEM can be guaranteed. With the above benefits, BEM has been successfully applied to the research of many problems, such as crack propagation, heat conduction, acoustics and so on. In reference [5], Yoshihiro used BEM to study the thermal conductivity of three-dimensional steady-state thermoelastic problems.

Isogeometric boundary element method (IGABEM) is a method that combines the isogeometric analysis (IGA) with BEM [6]. The main idea of IGA is that the polynomial interpolation function used in conventional FEM is replaced by the non-uniform rational B-spline (NURBS) basis function. During the numerical analysis, the IGA keeps the geometric model exact. Meanwhile, in IGA the geometric model and the mesh model share the same representation, which avoids the need for exchanging geometric data between different models during the analysis. Therefore, the IGA bridges the gap between CAD and CAE. IGABEM combines the advantages of the IGA and BEM. This method not only guarantees the accuracy of the geometric model during analysis, but also keeps the advantages of dimension reduction and high computational accuracy of BEM. Currently, IGABEM has been widely used in the fields of fluid problems, crack problems, heat transfer problems, elastic problems and so on. In reference [7], Yu et al. applied the IGABEM to study the heat transfer problem of integrated circuit packaging structures. Authors established a DBC model and compared the obtained numerical results with the reference solutions, which confirmed the effectiveness of the IGABEM for its application in electronic packaging structures.

This article analyzes the transient heat conduction problems in electronic packaging structure by using the proposed IGABEM. In the calculation, the heat source and time will lead to the domain integrals. To ensure the advantage of boundary discretization, the article uses the radial integration method (RIM) to convert domain integrals into equivalent boundary integrals. This article uses time marching method to handle the time terms.

II. METHOD

A. Normalized boundary integral equation for transient heat conduction.

The governing equation for transient heat conduction with constant coefficient can be derived as follows:

$$k\frac{\partial}{\partial x_i}\left(\frac{\partial T(\boldsymbol{x},t)}{\partial x_i}\right)+b(\boldsymbol{x},t)=\rho c_p\frac{\partial T(\boldsymbol{x},t)}{\partial t}\qquad (t\ge t_0, x\in\Omega) \tag{1}$$

where $\boldsymbol{k}$ is the thermal conductivity. $T(\boldsymbol{x}, t)$ is the temperature of point $\boldsymbol{x}$ at time t. $b(\boldsymbol{x}, t)$ is the heat source, which is a coordinate function at point $\boldsymbol{x}$. $\boldsymbol{\rho}$ and $\boldsymbol{c}_p$ denote respectively the density and specific heat of the material.

The boundary integral equation of transient heat conduction problems in domain Ω can be derived as follows:

$$\begin{aligned}c(p)\tilde{T}(p,t)=&-\int_\Gamma u^*(Q,p)q(Q,t)d\Gamma(Q)-\int_\Gamma q^*(Q,p)\tilde{T}(Q,t)d\Gamma(Q)\\&+\int_\Omega u^*(q,p)b(q,t)d\Omega(q)-\int_\Omega u^*(q,p)\tilde{\rho}(q)\dot{\tilde{T}}(q,t)d\Omega(q)\end{aligned} \tag{2}$$

where u^* and q^* are the fundamental solutions, and can be defined as:

$$u^*=\frac{1}{2\pi}\ln\left(\frac{1}{r}\right) \tag{3}$$

$$q^*=\frac{\partial u^*}{\partial \mathbf{n}}=-\frac{1}{2\pi r}\frac{\partial r}{\partial \mathbf{n}} \tag{4}$$

in which r represents the distance between the source point $\boldsymbol{p}$ and field point $\boldsymbol{q}$, and can be obtained by the following equation.

$$r=\sqrt{\left(x^p-x^q\right)^2+\left(y^p-y^q\right)^2} \tag{5}$$

From (2), we can find that there are two domain integrals (the last two terms). The first domain integral is related to the heat source. The heat source $b(q, t)$ can be a constant or a function of coordinate and time in general. By using RIM, we can transfer this domain integral to the equivalent boundary integral accurately. The second domain integral is related to the time t. Since the integral kernels contain unknown values, we must express the unknown values in terms of the known basis functions. Then, the domain integral can be transformed to the boundary integral by using the RIM.

By using RIM, the obtained boundary integral is shown as follows:

$$\int_\Omega u^*(q,p)b(q,t)d\Omega(q)=\int_\Gamma\frac{1}{r^\alpha(Q,p)}\frac{\partial r}{\partial \boldsymbol{n}}F(Q,p)d\Gamma(Q) \tag{6}$$

where $F(Q, p)$ is the radial integral with the following expression.

$$F(Q,p)=\int_0^{r(Q,p)}u^*(q,p)b(q,t)r^\alpha(q,p)dr(q) \tag{7}$$

Here, $r(Q, p)$ represents the distance from the source point p to the boundary collocation point Q. $r(q,p)$ denotes the distance between the source point p and the field point q in the considered region.

The second domain integral can be expressed as

$$\begin{aligned}\int_\Omega u^*(q,p)\tilde{\rho}(q)\dot{\tilde{T}}(q,t)d\Omega(q)=&\alpha^A\int_\Gamma\frac{1}{r^\alpha(Q,p)}\frac{\partial r}{\partial \boldsymbol{n}}F^A(Q,p)d\Gamma(Q)\\&+\alpha^k\int_\Gamma\frac{r_{,k}}{r^\alpha(Q,p)}\frac{\partial r}{\partial \boldsymbol{n}}F^1(Q,p)d\Gamma(Q)\\&+\left(\alpha^k x_k^p+\alpha^0\right)\int_\Gamma\frac{1}{r^\alpha(Q,p)}\frac{\partial r}{\partial \boldsymbol{n}}F^0(Q,p)d\Gamma(Q)\end{aligned} \tag{8}$$

where

$$F^A(Q,p)=\int_0^{r(Q,p)}\tilde{\rho}u^*(q,p)\phi^A(R)r^\alpha(q,p)dr(q) \tag{9}$$

$$F^1(Q,p)=\int_0^{r(Q,p)}\tilde{\rho}u^*(q,p)r^{\alpha+1}(q,p)dr(q) \tag{10}$$

$$F^0(Q,p)=\int_0^{r(Q,p)}\tilde{\rho}u^*(q,p)r^\alpha(q,p)dr(q) \tag{11}$$

The radial integration in (9) is a function of R, where R represents the distance between the field point q and the application point A. Additionally, the field point q varies along the radial direction $r(Q, p)$. The relationship among them is illustrated in Fig.1. Based on Fig.1, we can derive the following relationship:

$$R=\sqrt{r^2+2sr+\bar{R}^2} \tag{12}$$

where

$$\begin{aligned}\bar{R}&=\sqrt{\bar{R}_i\bar{R}_i}\\\bar{R}_i&=x_i^p-x_i^A\\s&=r_{,i}\bar{R}_i\end{aligned} \tag{13}$$

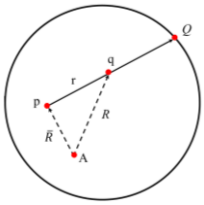


Fig.1. Distance relationship between source point p, field point q and application point A.

By submitting the obtained boundary integral (6) and (8) into (2), we can obtain the pure boundary integral equation. To solve this obtained boundary integral equation, the boundary Γ of computational domain needs to be discretized into boundary elements, and the Gauss quadrature method is used to compute the regular boundary integral on each boundary element. Assuming that the boundary is discretized into N_e boundary elements. To compute the domain integrals, we should configure N_b points on the boundary and N_i points within the domain. By evaluating (8) at each source point, we can obtain the following system of equations in matrix form:

$$\boldsymbol{H\tilde{T}-Gq=y_b-C\dot{\tilde{T}}} \tag{14}$$

where $\boldsymbol{H}$ and $\boldsymbol{G}$ are coefficient matrices that contain the values of integrals with kernels u^* and q^*, respectively. The column vector $\boldsymbol{q}$ contains the heat fluxes at the boundary nodes, and y_b is a column vector formed by the domain integration (6) due to the heat sources.

B. Implementation of radial integral method in BEM [8,9]

Define a Cartesian coordinate system and a polar coordinate system for a two-dimensional domain Ω

surrounded by boundary Γ. The relationships between the Cartesian and polar coordinate systems can be expressed as follows:

$$\begin{cases} r_1 = x_1 - x_1^p = r\cos\theta \\ r_2 = x_2 - x_2^p = r\sin\theta \end{cases} \quad 0 \le \theta \le 2\pi \tag{15}$$

where x_p represents the Cartesian coordinates at the source point p and r is the distance between the source point and a field point. In the polar coordinate system, a differential domain dΩ can be expressed as:

$$d\Omega = rdrd\theta \tag{16}$$

When the field point is located on the boundary, as shown in Fig. 2, the following relationship can be found.

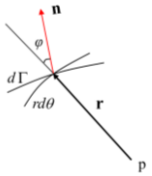


Fig.2. Relationship between differential elements $rd\theta$ and $d\Gamma$

$$rd\theta = d\Gamma\cos\varphi = d\Gamma\frac{r_i n_i}{r} \tag{17}$$

where φ is the angle formed by the differential arc $rd\theta$ with radius r and the differential boundary $d\Gamma$ with outward normal n_i. By using (16), the equation (17) can be written for boundary field points as:

$$d\Omega = rdrdS_I \tag{18}$$

where

$$dS_I = \frac{1}{r}\frac{\partial r}{\partial n}d\Gamma \tag{19}$$

Equation (18) is used to transform a domain integral to a boundary. Considering a general function $f(x)$ with x representing (x_1, x_2), its domain integral can be performed using (18) as follows:

$$\int_\Omega f(x)d\Omega = \int_s\int_0^{r(Q)} frdrdS_I(Q) = \int_S F(Q)sS_I(Q) \tag{20}$$

where

$$F(Q) = \int_0^{r(Q)} frdr \tag{21}$$

Substituting (19) into (20) leads to

$$\int_\Omega f(x)d\Omega = \int_\Gamma \frac{1}{r}\frac{\partial r}{\partial n}F(Q)d\Gamma(Q) \tag{22}$$

Based on the radial integral (21), the domain integral has been transformed into a boundary integral. It should be noted that the boundary integral in (22) is calculated in Cartesian coordinates, and the source point can be either a boundary node or an internal point.

C. *Implementation of time marching method for the time derivative term.*

The time derivative of the temperature is calculated by the forward difference method, i.e.

$$\frac{\partial\tilde{T}}{\partial t} = \frac{\tilde{T}^{n+1} - \tilde{T}^n}{\Delta t} \tag{23}$$

where $\tilde{T}^{n+1}$ represents the temperature at the n+1-th time step. The other physical quantities in (14) can be obtained by the values at the n-th and n+1-th time steps.

$$\tilde{T} = \theta\tilde{T}^{n+1} + (1-\theta)\tilde{T}^n \tag{24}$$

$$\tilde{q} = \theta\tilde{q}^{n+1} + (1-\theta)\tilde{q}^n \tag{25}$$

Substituting (23), (24) and (25) into (14), we obtain.

$$M\tilde{T}^{n+1} - \theta Gq^{n+1} = y_b - N\tilde{T}^n + (1-\theta)Gq^n \tag{26}$$

where

$$M = \theta H + C/\Delta t \tag{27}$$

$$N = (1-\theta)H - C/\Delta t \tag{28}$$

When the unknown quantity at the n+1-th time step is calculated, all the temperature and heat flux at the n-th step can be obtained, so (26) can be further rewritten as

$$M\tilde{T}^{n+1} - \theta Gq^{n+1} = y^n \tag{29}$$

where y^n is the sum of the terms at the right-hand side of (26).

Substituting the boundary conditions of time step n+1 into (29), the following system of algebraic equations is formed.

$$Ax^{n+1} = y^{n+1} \tag{30}$$

where $\boldsymbol{x}^{n+1}$ is column vector composed of unknown boundary temperature, boundary heat flux and internal point temperature at the n+1-th time step.

III. RESULTS AND DISCUSSIONS

To illustrate the accuracy and convergence of results obtained by the proposed method, two kinds of relative error are defined in (31) and (32), i.e. the relative error (RE)

$$RE = |T_{num,i}(x,t) - T_{Anal,i}(x,t)|/T_{Anal,i}(x,t) \tag{31}$$

and the mean relative error (MRE)

$$MRE = \frac{1}{N}\sum_{i=1}^{N}\left|T_{num,i}(x,t) - T_{Anal,i}(x,t)\right|/T_{Anal,i}(x,t) \tag{32}$$

where $T_{num,i}(x, t)$ is the temperature calculated by the proposed IGABEM, and $T_{Anal,i}(x, t)$ is the analytical result.

A. *Temperature distribution of a chip-Copper pillar bump structure*

In order to check the correctness of the present IGABEM, a chip-Copper pillar bump structure which is an important technology in the electronic packaging is analyzed. The geometry parameters of the considered model are given in Fig. 3 and detailed parameters are shown in Table Ⅰ.

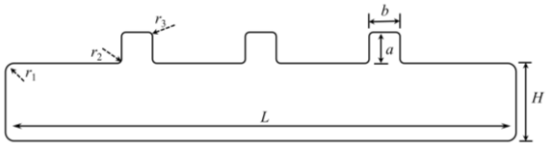


Fig.3. The geometry and the dimension of the model

TABLE Ⅰ PARAMETERS OF CHIP-COPPER PILLAR BUMP STRUCTURE

parameter	L	H	a	b	r_1	r_2	r_3
size(mm)	6.6	1	0.4	0.4	0.15	0.03	0.07

The control points and collocation points used in this model are shown Fig. 4. The boundary geometry of the chip-copper pillar bump structure is described by NURBS and the order (p) of basis functions is 2.

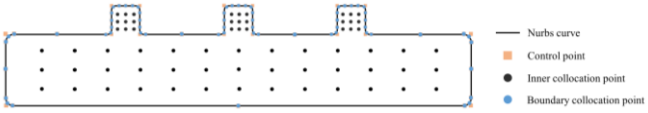


Fig.4. The control points and boundary collocation points used in the model

Here, we will calculate the temperature distribution in the model from 0 to 2s and set the heat source of the model as follows:

$$T(x,t)=x_1^2+x_2^2-4t \tag{33}$$

In this example, analytical results are employed to examine the computational results of IGABEM. The analytical result of the problem can be written as follows:

$$T(x,t)=(x_1^2+x_2^2)t \tag{34}$$

Fig.4 shows us the distribution of collocation points. Temperatures at five collocation points are computed corresponding to coordinates P_A (3.3,0.8), P_B (2.8,0.5), P_C (3.3,0.5), P_D(3.8,0.5), P_E(3.3,0.2). Fig. 5 gives us the relative errors of the computed result, from which we can find that the RE remains below 1.19×10^{-3}.

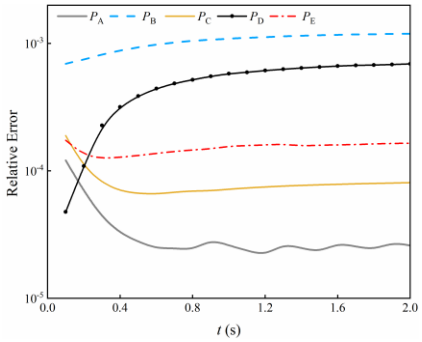


Fig.5. The relative errors of temperature for the considered collocation points

Table Ⅱ presents the numerical results at the point P_C for different time steps. From the table, we can find that the RE is extremely small, remaining at the order of 10^{-5}. Furthermore, with the time increasing, the relative error gradually stabilizes (ultimately reaching 5.3860×10^{-5}). In Fig 6, the temperature distribution obtained by the current numerical scheme and analytical method are given when t=2s. Based on the above analysis, we can conclude that the proposed method exhibits high accuracy and is suitable for analyzing heat transfer problems in electronic packaging.

TABLE Ⅱ COMPARISONS OF TEMPERATURE RESULT OBTAINED BY TWO METHODS

Time step t	Numerical result	Analytical result	Relative Error
0.5s	5.57031	5.57	5.5655E-05
1.0s	11.14060	11.14	5.3860E-05
1.5s	16.71090	16.71	5.3860E-05
2.0s	22.28120	22.28	5.3860E-05

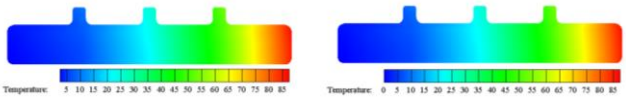
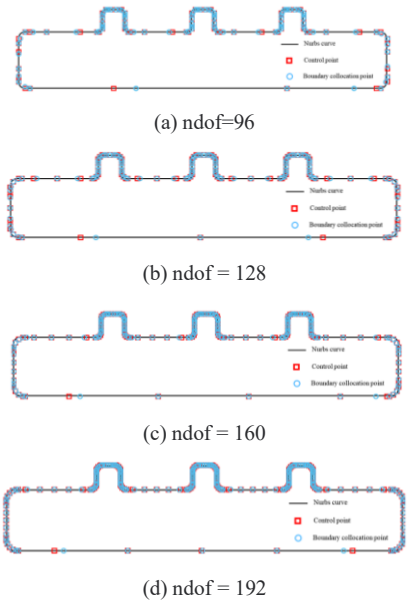


Fig.6. Temperature distributions obtained by the current scheme (left) and analytical method (right) when t = 2s

B. *The convergence of the current method*

To further study the convergence of the method, different numbers of degree of freedom (ndofs) are adopted to study the model. Fig. 7 shows the positions of the control point and boundary collocation points for the model under different ndofs .



(a) ndof=96

(b) ndof = 128

(c) ndof = 160

(d) ndof = 192

Fig.7. The control points and boundary collocation points with different ndofs.

Fig.8 presents the RE at point P_E under different ndofs. As the ndof increases from 96 to 192, the convergence can be seen clearly.

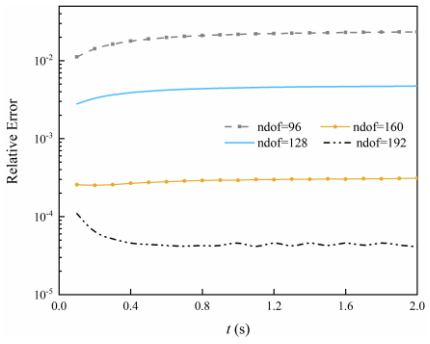


Fig.8. The relative error at Point P_E under different ndofs.

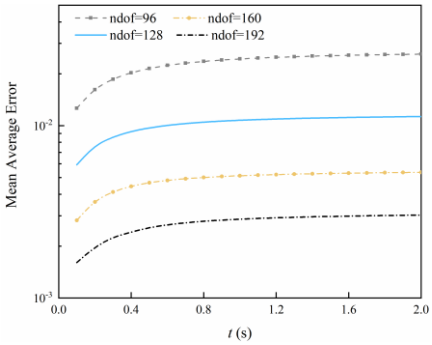


Fig.9. The mean relative error of collocation points under different degrees of freedom.

Fig.9 illustrates the MRE of temperature at the studied collocation points under different ndofs. A similar conclusion can be drawn as the above RE result. the MRE of temperature obtained by the proposed scheme when ndof = 96 are unsatisfactory, but with the refinement of knot spans the proposed method can yield accurate results.

IV. CONCLUSION

This paper proposes a transient IGABEM to solve transient heat conduction problems. The method uses RIM and time marching method to solve domain integrals and time-dependent partial differential equations, respectively. By solving the obtained linear system equation, the temperatures at each time step domain can be obtained. It was found that as the number of degree of freedom increases, the accuracy of the computed results improves. At the same time, numerical example demonstrates that the IGABEM can accurately solve transient heat conduction problems in electronic packaging.

ACKNOWLEDGMENT

This research is supported by the National Natural Science Foundation of China (No. 12002009), R&D Program of Beijing Municipal Education Commission (No. KM202110005032) and "QiHang Programme" for Faculty of Materials and Manufacturing, BJUT (QH202303).

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